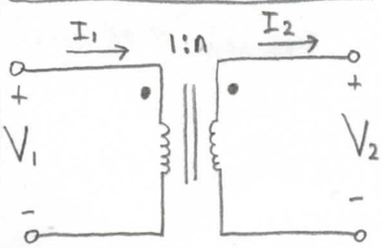
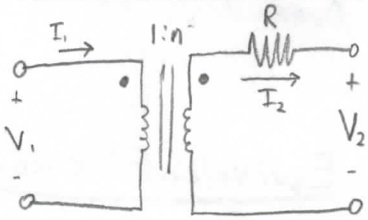
ELEC 342

Ideal Transformer

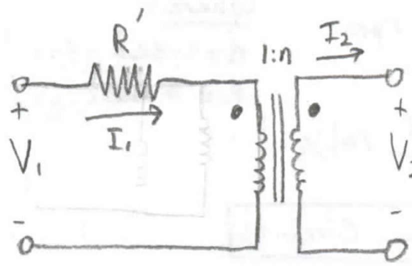


① $V_2 = nV_1$ ② $I_1 = nI_2$

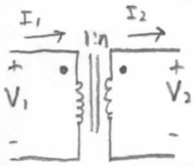
Transforming Impedances



$R' = \frac{R}{n^2} \longrightarrow$

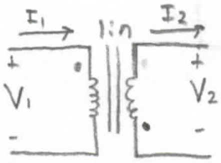


Dot Convention



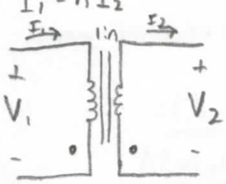
$V_2 = nV_1$

$I_1 = nI_2$



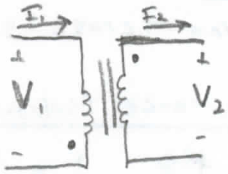
$V_2 = -nV_1$

$I_1 = -nI_2$



$V_2 = nV_1$

$I_1 = nI_2$



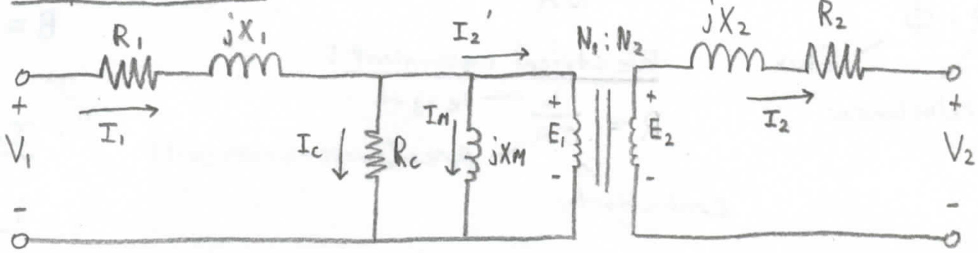
$V_2 = -nV_1$

$I_1 = -nI_2$

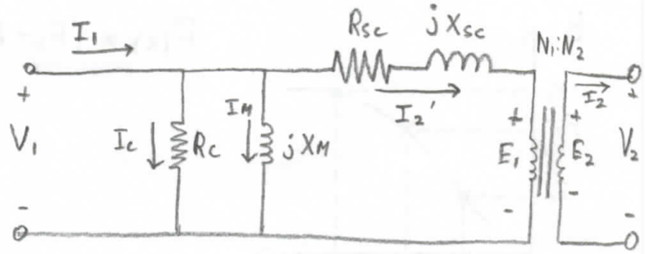
"whatever spot the current on the primary enters (Dot or non-Dot) that will be the higher voltage on the secondary (Dot or non-Dot)"

Ideal Transformer

Complete Model:

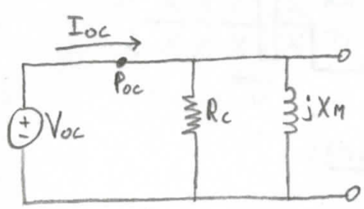


Simplified Model:



Open Circuit Test

- Determines R_c, X_M
- Apply rated Voltage to LVS

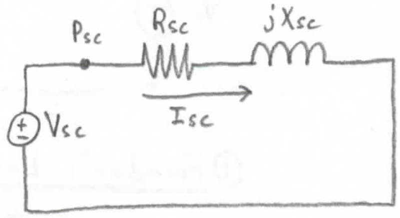


① $R_c = \frac{V_{oc}^2}{P_{oc}}$ • $I_c = \frac{V_{oc}}{R_c}$

② $X_M = \frac{V_{oc}}{I_M}$ • $I_M = \sqrt{I_{oc}^2 - I_c^2}$

Short Circuit Test

- Determines R_{sc}, X_{sc}
- Apply rated current to HVS



① $R_{sc} = \frac{P_{sc}}{I_{sc}^2}$

② $X_{sc} = \sqrt{\left(\frac{V_{sc}}{I_{sc}}\right)^2 - R_{sc}^2}$

Induction Motors

Rotor Speed / Torque:

$$n_{stator} = \frac{120f}{P} \text{ rpm}$$

where:
n = speed
P = # Poles/2

$$\omega = \frac{2\pi f}{P} \text{ rad/s}$$

Slip:

"Rotor follows the speed of the RMF"

$$S = \left(\frac{n_{RMF} - n_{rotor}}{n_{RMF}}\right) \cdot 100\%$$

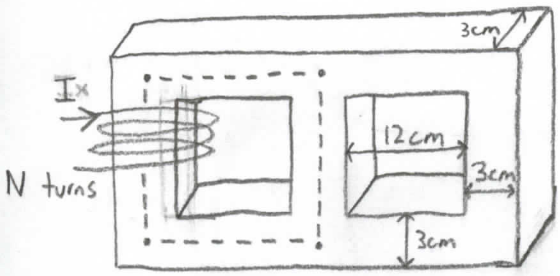
$$n_{rotor} = (1-S)\, n_{RMF}$$

4.44 Law

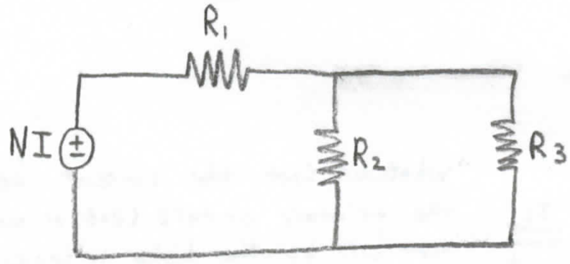
$$E_{rms} = 4.44 f N \Phi_{peak}$$

E_{rms} – induced voltage, f – frequency, N – # of turns, Φ_{peak} – flux

Magnetic Circuits



Equivalent circuit:



Ohm's Law:

$$\mathcal{U} = \mathcal{R} \cdot \Phi$$

$\mathcal{U}$ – mmf, $\mathcal{R}$ – reluctance, Φ – flux

$$\mathcal{R} = \frac{L}{\mu A} \text{ (assuming constant } \mu, A)$$

Electrical equivalent:

$$R = \frac{L}{\sigma A}$$

L – length, A – Area (cross-sectional), σ – conductivity

Flux Density: (B) $^*B = \mu_0 \mu_r H$

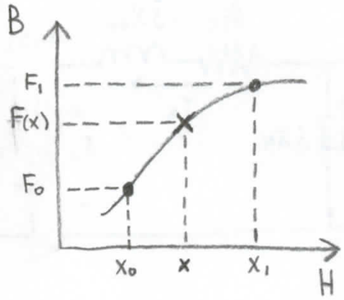
$$B = \frac{\Phi}{\text{Area}}$$

Φ – flux, Area – cross-sectional area

Inductance (solenoid):

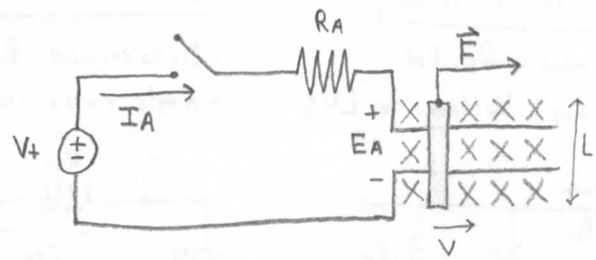
$$L = \frac{N\Phi}{I}, \quad L = \frac{\mu_0 N^2 A}{l}$$

Logarithmic Interpolation



$$F(x) = (F_1 - F_0)\left[\frac{\log\left(\frac{x}{x_0}\right)}{\log\left(\frac{x_1}{x_0}\right)}\right] + F_0$$

Linear DC Machine



① **Faraday's Law (Motor):**

$$E_A = L \cdot (\vec{V} \times \vec{B}) = BLV$$

② **Lorentz Force Law (Generator):**

$$\vec{F} = I(\vec{L} \times \vec{B}) = BIL$$

③ **Kirchoff's Voltage Law:**

$$V_+ - I_A R_A - E_A = 0$$

Linear DC Machine states

① **Start up:** $(E_a = 0)$

$$I_A = \frac{V_+}{R_A}, \quad F = BIL$$

② **No load steady state:** $(I_A = 0)$

$$E_A = V_+, \quad E_A = BLV_{ss}$$

③ **load** $(I_A \neq 0)$

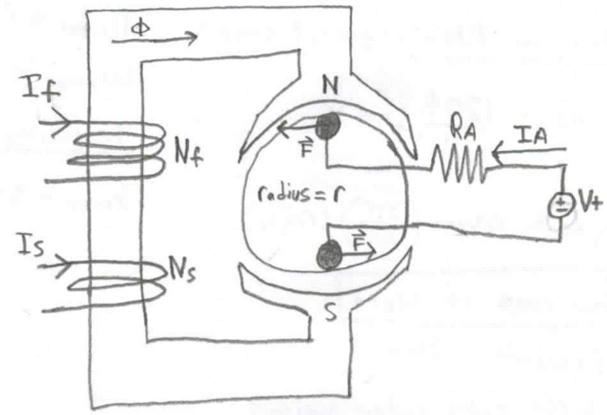
$$I_A = \frac{F_{Load}}{BL}, \quad E_A = V_+ - I_A R_A, \quad E_A = BLV_{ss}$$

torque:

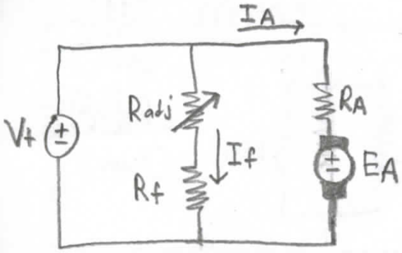
$$\tau = \frac{MR_a}{(Bl)^2}$$

Motor Excitations

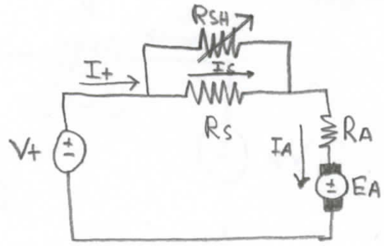
$mmf = N_f I_f + N_s I_s$



① Shunt:



② Series:



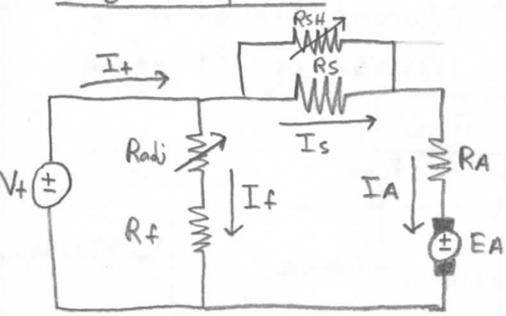
$T = 2rFN = 2rBILN = 2Nr\frac{\Phi}{A}LI$

$= k\phi I_A$

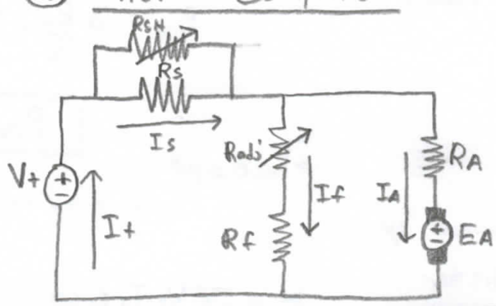
where:

$k = \frac{2NrL}{A}$

③ Long-Compound:



④ Short-Compound:



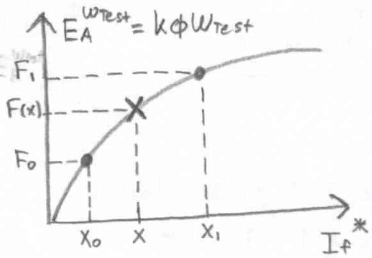
Power Flow

$P_{out} = P_{conv} - P_{losses}$

$P_{conv} = E_A I_A$

$P_{out} = T_{out} W_{mech}$

DC motor Magnetization Curve



Coffee Formulas:

$E_A = k\phi W$

$T_{ind} = k\phi I_A$

$I_f^* = \frac{N_f I_f + N_s I_s}{N_f}$

Two Eqn's, Two Unknowns

① $P_{out} = E_A I_A - P_{losses}$

② $V_t - I_A R_A - E_A = 0$

Armature Reaction

*compensation windings means no armature reaction

$I_f^* = I_f + I_s\left(\frac{N_s}{N_f}\right) - \underbrace{k_{Ar}^* I_A}_{\text{armature reaction}}$

$k_{Ar}^* = \frac{k_{Ar}}{I_A}$ *test, given

$\frac{E_{A1}}{W_1} = \frac{E_{A2}}{W_2}$, $\therefore W_2 = \left(\frac{E_{A2}}{E_{A1}}\right)W_1$

Linear Interpolation:

$F(x) = (F_1 - F_0)\left(\frac{X - X_0}{X_1 - X_0}\right) + F_0$

** IND motor (to find R_2)

$V_{th} = V_1$(opening Z_2)

Z_{th} = "looking back from Z_2"

$R_{th} = Re(Z_{th})$

$X_{th} = Im(Z_{th})$

$$\frac{P_{conv}}{W_{rotor}} = \frac{3}{W_{sync}} \cdot \frac{V_{th}^2 \cdot (R_2/s)}{\left[(R_{th} + R_2/s)^2 + (X_{th} + X_2)^2\right]} = T_{ind}$$

Induction Motor

Creating RMF: (squirrel cage)

$$n_{sync} = \frac{120f}{P} \text{ (rpm)}$$

$$\omega_{sync} = n_{sync}\left(\frac{2\pi}{60}\right) \text{ (rad/s)}$$

How does it Work

1. Generate RMF
2. RMF cuts rotor wires
3. Induced voltage in wire (Current flows)
4. Force on wires = torque
5. Rotor starts rotating

Induced Torque

$$T_{ind} = \frac{P_{AG}}{\omega_{sync}} \rightarrow P_{AG} = \text{Power drained by } R_2/s \text{ resistor}$$

$$T_{ind} = \frac{P_{conv}}{\omega_{rotor}}$$

States:

1. **max torque:** $\frac{R_2}{s} = |Z_{th}|$ "looking back"
2. **Startup:** Set $s=1$, resolve for V_1, I_2 and calculate P_{GAP} which is now just drained by R_2 (ie: $s=1$)

Rotor Slip (% difference in speed) (1/3 Phase)

$$n_{rotor} = (1-s)\, n_{sync}$$

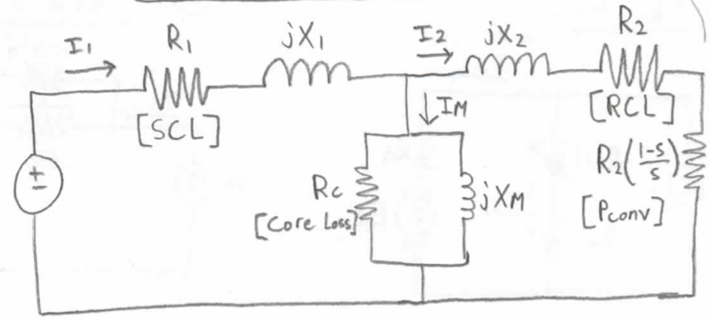
$$\omega_{rotor} = (1-s)\,\omega_{sync}$$

frequency:

$$f_{rotor} = s f_{sync}$$

Referred to stator side

Equivalent Circuit



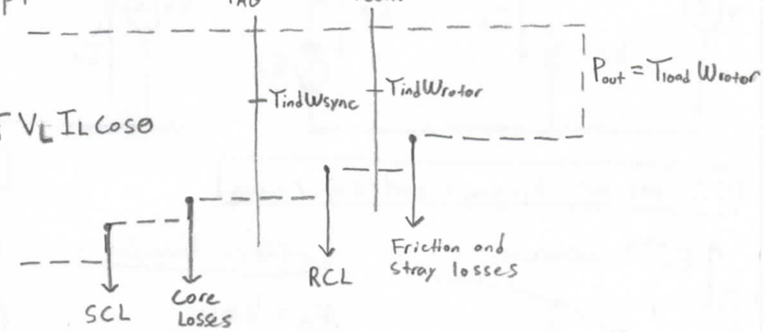
Gap Power:

$$P_{gap} = 3\,\frac{R_2}{s}\cdot I_2^2 \text{ (Current drained by } \tfrac{R_2}{s} \text{ resistor in 1/3 phase)}$$

Power Flow

$*\cos\theta = pf$

$$P_{in} = \sqrt{3}\, V_L I_L \cos\theta$$



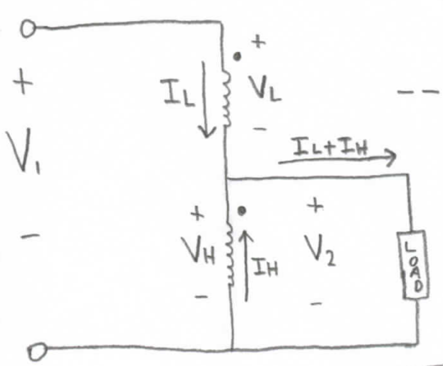
Example:

"A 250V, 85hz, 70hp, three phase Ind motor has a power factor of 0.8 lagging and draws 45A

$V_L = 480V$
$I_L = 45A$
$pf = \cos\theta = 0.8$

Autotransformer

"A 27.5kVA 2200/680V"



$S_T = 27.5\text{kVA}$
$V_H = 2200V$
$V_L = 680V$

$$I_L = \frac{S_T}{V_L}, \quad I_H = \frac{S_T}{V_H}$$

$$S_{Load} = \left(\frac{V_H + V_L}{V_L}\right) S_T$$

"ie: much larger"

$$R_c = \frac{V_o^2}{P_o}, \quad X_M = \frac{V_o}{\sqrt{I_o^2 - \left(\frac{V_o}{R_c}\right)^2}}$$

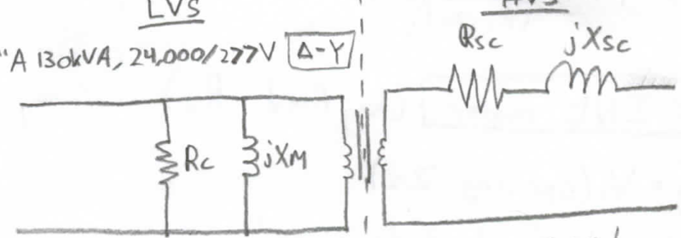
$$R_{sc} = \frac{P_{sc}}{I_{sc}^2}, \quad X_{sc} = \sqrt{\left(\frac{V_{sc}}{I_{sc}}\right)^2 - R_{sc}^2}$$

(Get everything per phase)

Transformer Parameters (P.U.)

LVS | HVS

Ex: "A 130kVA, 24,000/277V $\boxed{\Delta - Y}$



LVS — measured: V_o, I_o, P_o

$S_{Base} = 130k$

$V_{LBase} = 277V$

$$I_{LBase} = \frac{S_{BASE}}{V_{LBASE}}$$

$$Z_{LBase} = \frac{V_{LBase}}{I_{LBase}}$$

HVS — measured: V_{sc}, I_{sc}, P_{sc}

$S_{Base} = 130k$

$V_{HBase} = 24kV$

$$I_{HBase} = \frac{S_{Base}}{V_{HBase}}$$

$$Z_{HBase} = \frac{V_{HBase}}{I_{HBase}}$$

Single Phase and Special Purpose Motors

Why: "common household devices don't use three phase and don't generate an RMF" [Pulse]

Starting Single Phase induction motors:

1. Split-Phase windings
2. Capacitor-type windings
3. Shaded stator windings

Capacitor Start

1. Expensive, give high starting torque

"Capacitor placed in series with the auxillery winding"

Different types of motors

" Can accelerate up to nsync by themselves"

1. Reluctance Motor:

"Relies on induced reluctance torque from iron for operation, orients itself to line up with external magnetic field

2. Hysteresis Motor:

"RMF magnetizes the metal of the rotor and induces poles within it"

3. Stepper Motor:

"Rotates a specific number of degrees discretely based on pulses from the source"

1. Permament magnet type
2. Reluctance type

"Useful in control and positioning systems

4. Brushless Motor:

"Brush wear and sparking is a real problem for maintenance on small DC machines"

- Similar to stepper but include a position sensor

1. High Price
2. Low maintenance
3. Long lifespan

Universal Motor

"Take a series DC motor and apply an AC source voltage instead of DC"

Characteristics:

1. poles and frame have to be laminated [otherwise enormous core losses]
2. Good for light weight and high torque applications
3. Examples: Vaccum cleaner, drill, blenders etc

Conditions for Parallel Operation

(3ϕ Transformers)

1. Same ratio
2. Same clock number
3. same short circuit impedance

Synchronous Vs. Induction Motor

IND has exact same stator, but different rotor (Cage or Wound)

Why have a squirrel cage on a SYNCH

"To start the motor, RMF rotating too fast"

Downside of DC motors

- Maintenance

Gap Power

"Power crossing the gap from stator to rotor"

$P_{AG} = \tau_{ind}\omega_{sync}$

of slip rings

Slip rings

- 2 [SYNCH]
- 3 [Induction]

Round vs Salient poles

round $\longrightarrow$ High speed

Salient $\longrightarrow$ More poles